Journal of the Egyptian Mathematical Society (2013) **21**, 75–78

Egyptian Mathematical Society

Journal of the Egyptian Mathematical Society

www.etms-eg.org
www.elsevier.com/locate/joems

ORIGINAL ARTICLE

Generalized derivations with power values in rings and Banach algebras

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Received 7 March 2012; revised 26 September 2012; accepted 3 January 2013
Available online 1 March 2013

KEYWORDS

Prime and semiprime ring;
Generalized derivation;
Lie ideal;
Banach algebra

Abstract Let R be a 2-torsion-free prime ring with center $Z(R)$, F a generalized derivation associated with a nonzero derivation d, L a Lie ideal of R. If $(d(u)^{l_1}F(u)^{l_2}d(u)^{l_3}F(u)^{l_4}\dots F(u)^{l_k})^n = 0$ for all $u \in L$, where $l_1, l_2, \dots, l_k$ are fixed non-negative integers not all zero, and n is fixed positive integer, then $L \subseteq Z(R)$. We also examine the case when R is a semiprime ring. Finally, we apply the above result to Banach algebras.

2000 MATHEMATICS SUBJECT CLASSIFICATION: 16N60, 16W25, 46K15, 47B47

1. Introduction

In all that follows, unless stated otherwise, R will be an associative ring, $Z(R)$ the center of R, Q its Martindale quotient ring and U its Utumi quotient ring. The center of U, denoted by C, is called the extended centroid of R (we refer the reader to [1] for these objects). By a Banach algebra we shall mean complex normed algebra A whose underlying vector space is a Banach space. The Jacobson radical $rad(A)$ of A is the intersection of all primitive ideals. If the Jacobson radical reduces to the zero element, A is called semisimple. For any $x, y \in R$, the symbol $[x, y]$ denotes the Lie product $xy - yx$. A ring R is called 2-torsion free, if whenever $2x = 0$, with $x \in R$, then $x = 0$. Recall that a ring R is prime if for any $a, b \in R$, $aRb = (0)$ implies $a = 0$ or $b = 0$, and is semiprime if for any $a \in R$, $aRa = (0)$ implies $a = 0$. An additive mapping $d: R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. In particular d is an inner derivation induced by an element $a \in R$, if $d(x) = [a, x]$ for all $x \in R$. The standard identity s_4 in four variables is defined as follows:

$$s_4 = \sum 1)^{\tau} X_{\tau(1)} X_{\tau(2)} X_{\tau(3)} X_{\tau(4)}$$

where $(-1)^{\tau}$ is the sign of a permutation τ of the symmetric group of degree 4.

Let us introduce the background of our investigation. Singer and Werner [2] obtained a fundamental result which stated investigation into the ranges of derivations on Banach algebras. In [2], Singer and Werner proved that any continuous derivation on a commutative Banach algebra has the range in the Jacobson radical of the algebra. In this paper they conjectured that the continuity is not necessary. Thomas [3] verified this conjecture. It is clear that the same result of Singer and Werner does not hold in noncommutative Banach

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Peer review under responsibility of Egyptian Mathematical Society.

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http://dx.doi.org/10.1016/j.joems.2013.01.001

algebras because of inner derivations. Hence in this context a very interesting question is how to obtain noncommutative version of Singer-Werner theorem. A first answer to this problem has been obtained by Sinclair in [4]. He proved that every continuous derivation of a Banach algebra leaves primitive ideals of the algebra invariant. In [5], Kim proved that if a noncommutative Banach algebra A admits a continuous linear Jordan derivation d such that $d(x)[d(x), x]d(x) \in rad(A)$ for all $x \in A$ then $d(A) \subseteq rad(A)$. More recently, Park [6] proved that if d is a linear continuous derivation of a noncommutative Banach algebra A such that $[[d(x), x], d(x)] \in rad(A)$ for all $x \in A$ then $d(A) \subseteq rad(A)$. In [7], Filippis extended the Park's result to generalized derivations. In the meanwhile many authors obtained more information about derivations satisfying certain suitable conditions in Banach algebra. For example, in [8] Vukman proved that if d is a linear derivation of a noncommutative semisimple Banach algebra A such that $[d(x), x]d(x) = 0$ for all $x \in A$, then $d = 0$.

In [9], Brešar introduced the definition of generalized derivation: an additive mapping $F{:}R \to R$ is called a generalized derivation if there exists a derivation $d{:}R \to R$ such that $F(xy) = F(x)y + xd(y)$ holds for all $x, y \in R$, and d is called the associated derivation of F. Hence, the concept of generalized derivations covers both the concepts of a derivation and of a left multiplier (i.e., an additive mapping satisfying $f(xy) = f(x)y$ for all $x, y \in R$). Basic examples are derivations and generalized inner derivations (i.e., mappings of type $x \to ax + xb$ for some $a, b \in R$). We refer to call such mappings generalized inner derivations for the reason they present a generalization of the concept of inner derivations. In [10], Hvala studied generalized derivations in the context of algebras on certain norm spaces. In [11], Lee extended the definition of a generalized derivation as follows: by a generalized derivation we mean an additive mapping $F{:}I \to U$ such that $F(xy) = F(x)y + xd(y)$ holds for all $x, y \in I$, where I is a dense left ideal of R and d is a derivation from I into U. Moreover, Lee also proved that every generalized derivation can be uniquely extended to a generalized derivation of U and thus all generalized derivations of R will be implicitly assumed to be defined on the whole of U. Lee obtained the following: every generalized derivation F on a dense left ideal of R can be uniquely extended to U and assumes the form $F(x) = ax + d(x)$ for some $a \in U$ and a derivation d on U.

On the other hand, a well-known result of Herstein [12] states that if ρ is a right ideal of R such that $u^n = 0$ for all $u \in \rho$, where n is a fixed positive integers, then $\rho = 0$. In [13], Chang and Lin considered the situation when $d(u)u^n = 0$ for all $u \in \rho$, where d is a nonzero derivation of R. In [14], Dhara and De Filippis studied the case when $u^s H(u) u^t = 0$ for all $u \in L$, where L a noncommutative Lie ideal of R, H a generalized derivation of R and s, t are fixed nonnegative integers. More precisely, they proved the following: Let R be a prime ring, H a nonzero generalized derivation of R and L a noncommutative Lie ideal of R. Suppose that $u^s H(u) u^t = 0$ for all $u \in L$. Then R satisfies s_4, the standard identity in four variables.

The present paper is motivated by the previous results and we here continue this line of investigation by examining what happens a ring R (or an algebra A) satisfying the identity $(d(u)^{l_1} F(u)^{l_2} d(u)^{l_3} F(u)^{l_4} \ldots F(u)^{l_k})^n = 0$ for all u in some appropriate subset of R (or A).

2. The results

Theorem 2.1. *Let R be a 2-torsion-free prime ring with center $Z(R)$, F a generalized derivation associated with a nonzero derivation d, L a Lie ideal of R. If $(d(u)^{l_1} F(u)^{l_2} d(u)^{l_3} F(u)^{l_4} \ldots F(u)^{l_k})^n = 0$ for all $u \in L$, where $l_1, l_2, \ldots, l_k$ are fixed nonnegative integers not all zero, and n is fixed positive integer, then $L \subseteq Z(R)$.*

Proof. Suppose that $L \not\subseteq Z(R)$. Since R is a prime ring and F is a generalized derivation of R, by Lee [11, Theorem 3], $F(x) = ax + d(x)$ for some $a \in U$. Since $\mathrm{Char}(R) \neq 2$ it follows from Herstein [12, pp.4-5], that there exists a nonzero two-sided ideal I of R such that $0 \neq [I, R] \subseteq L$. In particular, $[I, I] \subseteq L$, hence without loss of generality we may assume that $L = [I, I] \subseteq L$. By the given hypothesis we have $(d([x,y])^{l_1} F([x,y])^{l_2} d([x,y])^{l_3} F([x,y])^{l_4} \ldots F([x,y])^{l_k})^n = 0$ for all $x, y \in I$. This implies that $(([d(x),y] + [x,d(y)])^{l_1} (a[x,y] + [d(x),y] + [x,d(y)])^{l_2} \ldots (a[x,y] + [d(x),y] + [x,d(y)])^{l_k})^n = 0$ for all $x, y \in I$. By Kharchenko [15], we divide the proof into two cases:

Case 1. Let d be an outer derivation of U, then R satisfies the polynomial identity $(([s,y] + [x,t])^{l_1} (a[x,y] + [s,y] + [x,t])^{l_2} \ldots (a[x,y] + [s,y] + [x,t])^{l_k})^n = 0$ for all $x, y, s, t \in I$. In particular, for $x = 0$, we arrive at $[s,y]^p = 0$ for all $s, y \in I$, where $p = n(l_1 + l_2 + \cdots + l_k)$, and by Herstein [16, Theorem 2], R is commutative, a contradiction.

Case 2. Let now d be the inner derivation induced by an element $q \in Q$, that is $d(x) = [q, x]$ for all $x, y \in U$. It follows that $(([[q,x],y] + [x,[q,y]])^{l_1} (a[x,y] + [[q,x],y] + [x,[q,y]])^{l_2} \ldots (a[x,y] + [[q,x],y] + [x,[q,y]])^{l_k})^n = 0$ for all $x, y \in I$. By Chuang [17, Theorem 2], I and Q satisfy the same generalized polynomial identities (GPIs), we have $(([[q,x],y] + [x,[q,y]])^{l_1} (a[x,y] + [[q,x],y] + [x,[q,y]])^{l_2} \ldots (a[x,y] + [[q,x],y] + [x,[q,y]])^{l_k})^n = 0$ for all $x, y \in Q$. In case center C of Q is infinite, we have $(([[q,x],y] + [x,[q,y]])^{l_1} (a[x,y] + [[q,x],y] + [x,[q,y]])^{l_2} \ldots (a[x,y] + [[q,x],y] + [x,[q,y]])^{l_k})^n = 0$ for all $x, y \in Q \otimes_C \overline{C}$, where $\overline{C}$ is the algebraic closure of C. Since both Q and $Q \otimes_C \overline{C}$ are prime and centrally closed [18, Theorem 2.5 and Theorem 3.5], we may replace R by Q or $Q \otimes_C \overline{C}$ according as C is finite or infinite. Thus we may assume that R is centrally closed over C (i.e. $RC = C$) which is either finite or algebraically closed and $(([[q,x],y] + [x,[q,y]])^{l_1} (a[x,y] + [[q,x],y] + [x,[q,y]])^{l_2} \ldots (a[x,y] + [[q,x],y] + [x,[q,y]])^{l_k})^n = 0$ for all $x, y \in R$. By Martindale [19, Theorem 3], RC (and so R) is a primitive ring which is isomorphic to a dense ring of linear transformations of a vector space V over a division ring D.

Assume that $\dim V_D \geqslant 3$.

First of all, we want to show that v and qv are linearly D-dependent for all $v \in V$. Since if $qv = 0$ then v, qv is D-dependent, suppose that $qv \neq 0$. If v and qv are D-independent, since $\dim V_D \geqslant 3$, then there exists $w \in V$ such that v, qv, w are also D-independent. By the density of R, there exists $x, y \in R$ such that: $xv = 0$, $xqv = w$, $xw = v$; $yv = 0$, $yqv = 0$, $yw = v$. These imply that $v = (([[q,x],y] + [x,[q,y]])^{l_1} (a[x,y] + [[q,x],y] + [x,[q,y]])^{l_2} \ldots (a[x,y] + [[q,x],y] + [x,[q,y]])^{l_k})^n v = 0v = 0$, which is a contradiction. So we conclude that v and qv are linearly D-dependent for all $v \in V$.

Our next goal is to show that there exists $b \in D$ such that $qv = vb$ for all $v \in V$. In fact, choose $v, w \in V$ linearly independent. Since $dim\ V_D \geqslant 3$, then there exists $u \in V$ such that u, v, w are linearly independent, and so $b_u, b_v, b_w \in D$ such that $qu = ub_u$, $qv = vb_v$, $qw = wb_w$, that is $q(u + v + w) = u b_u + vb_v + wb_w$. Moreover $q(u + v + w) = (u + v + w) b_{u+v+w}$ for a suitable $b_{u+v+w} \in D$. Then $0 = u(b_{u+v+w} - b_u) + v(b_{u+v+w} - b_v) + w(b_{u+v+w} - b_w)$ and because u, v, w are linearly independent, $b_u = b_v = b_w = b_{u+v+w}$, that is b does not depend on the choice of v. Hence now we have $qv = vb$ for all $v \in V$.

Now for $r \in R$, $v \in V$, we have $(rq)v = r(qv) = r(vb) = (rv)b = q(rv)$, that is $[q, R]V = 0$. Since V is a left faithful irreducible R-module, hence $[q, R] = 0$, i.e. $q \in Z(R)$ and so $d = 0$, a contradiction.

Suppose now that $dim\ V_D \leqslant 2$.

In this case R is a simple GPI-ring with 1, and so it is a central simple algebra finite dimensional over its center. By Lanski [20, Lemma 2], it follows that there exists a suitable filed F such that $R \subseteq M_k(F)$, the ring of all $k \times k$ matrices over F, and moreover $M_k(F)$ satisfies the same GPI as R.

Assume $k \geqslant 3$, by the same argument as in the above, we can get a contradiction.

Obviously if $k = 1$, then R is commutative, again a contradiction.

Thus we may assume that $k = 2$, i.e., $R \subseteq M_2(F)$, where $M_2(F)$ satisfies $((([[q,x],y]+[x,[q,y]])^{l_1}(a[x,y]+[[q,x],y]+[x,[q,y]])^{l_2}\ldots(a[x,y]+[[q,x],y]+[x,[q,y]])^{l}{}_{k})^n = 0$. Denote e_{ij} the usual matrix unit with 1 in (i, j)-entry and zero elsewhere. Let $[x, y] = [e_{21}, e_{11}] = e_{21}$. It is easy to see that $((qe_{21}-e_{21}q)^{l_1}(ae_{21}+qe_{21}-e_{21}q)^{l_2}\ldots(ae_{21}+qe_{21}-e_{21}q)^{l_k})^n = 0$. Right multiplication by e_{21} in the above equation gives that $(-1)^m(e_{21}q)^m e_{21} = ((qe_{21}-e_{21}q)^{l_1}(ae_{21}+qe_{21}-e_{21}q)^{l_2}\ldots(ae_{21}+qe_{21}-e_{21}q)^{l_k})^n e_{21} = 0e_{21} = 0$, where $m = n(l_1 + l_2 + \cdots + l_k)$. Set $q = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix}$, then by calculation we find that $(-1)^m \begin{pmatrix} 0 & 0 \\ q_{12}^m & 0 \end{pmatrix} = 0$, which implies that $q_{12} = 0$. Similarly we can see that $q_{21} = 0$. Therefore q is diagonal in $M_2(F)$. Let $f \in Aut(M_2(F))$. Since $((([f(q),f(x)],f(y)]+[f(x),[f(q),f(y)]])^{l_1}(f(a)[f(x),f(y)]+[[f(q),f(x)],f(y)]+[f(x),[f(q),f(y)]])^{l_2}\ldots(f(a)[f(x),f(y)]+[[f(q),f(x)],f(y)]+[f(x),[f(q),f(y)]])^{l}{}_{k})^n = 0$ so $f(q)$ must be a diagonal matrix in $M_2(F)$. In particular, let $f(x) = (1 - e_{ij})x(1 + e_{ij})$ for $i \neq j$, then $f(q) = q + (q_{ii} - q_{jj})e_{ij}$, that is $q_{ii} = q_{jj}$ for $i \neq j$. This implies that q is central in $M_2(F)$, which leads to $d = 0$, a contradiction. This completes the proof of the theorem. $\square$

The following example demonstrates that R to be prime is essential in the hypothesis of Theorem 2.1.

Example 2.2. Let Z be the ring of integers. Set $R = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} | a, b, c \in Z \right\}$ and $L = \left\{ \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} | b \in Z \right\}$. We define the following maps: $F\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} a & 2b \\ 0 & 0 \end{pmatrix}$. $d\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}$. Then it is easy to see that L is a Lie ideal and F is a generalized derivation associated with a nonzero derivation d of R. Moreover, it is straightforward to check that F satisfies the property $(d(u)^{l_1}F(u)^{l_2}d(u)^{l_3}F(u)^{l_4}\ldots F(u)^{l_k})^n = 0$ for all $u \in L$, where $l_1, l_2, \ldots, l_k$ and n are fixed positive integers, however $L \not\subseteq Z(R)$.

Corollary 2.3. *Let R be a 2-torsion-free prime ring, F a generalized derivation associated with a nonzero derivation d. If $(d(r)^{l_1}F(r)^{l_2}d(r)^{l_3}F(r)^{l_4}\ldots F(r)^{l_k})^n = 0$ for all $r \in R$, where $l_1, l_2, \ldots, l_k$ are fixed non-negative integers not all zero, and n is fixed positive integer, then R is commutative.*

Theorem 2.4. *Let R be a 2-torsion-free semiprime ring, F a generalized derivation associated with a nonzero derivation d. If $(d(r)^{l_1}F(r)^{l_2}d(r)^{l_3}F(r)^{l_4}\ldots F(r)^{l_k})^n = 0$ for all $r \in R$, where $l_1, l_2, \ldots, l_k$ are fixed non-negative integers not all zero, and n is fixed positive integer, then there exists a central idempotent element e in U such that on the direct sum decomposition $R = eU \oplus (1-e)U$, d vanishes identically on eU and the ring $(1-e)U$ is commutative.*

Proof. We are given that $(d(r)^{l_1}F(r)^{l_2}d(r)^{l_3}F(r)^{l_4}\ldots F(r)^{l_k})^n = 0$ for all $r \in R$. Since R is semiprime and F is a generalized derivation of R, by Lee [11, Theorem 3], $F(x) = ax + d(x)$ for some $a \in U$. And hence we have $(d(r)^{l_1}(ar+d(r))^{l_2}d(r)^{l_3}(ar+d(r))^{l_4}\ldots(ar+d(r))^{l_k})^n = 0$ for all $r \in R$. By Lee [22, Theorem 3], R and U satisfy the same differential identities, then $(d(r)^{l_1}(ar+d(r))^{l_2}d(r)^{l_3}(ar+d(r))^{l_4}\ldots(ar+d(r))^{l_k})^n = 0$ for all $r \in U$. Let B be the complete Boolean algebra of idempotents in C and M be any maximal ideal of B. Since U is a B-algebra orthogonal complete [21, p.42], and MU is a prime ideal of U, which is d-invariant. Denote $\overline{U} = U/MU$ and $\bar{d}$ the derivation induced by d on $\overline{U}$, i.e., $\bar{d}(\bar{u}) = \overline{d(u)}$ for all $u \in U$. For all $\bar{r}, \in \overline{U}$, $(\bar{d}(\bar{r})^{l_1}(\bar{a}\bar{r}+\bar{d}(\bar{r}))^{l_2}\bar{d}(\bar{r}^{l_3}(\bar{a}\bar{r}+\bar{d}(\bar{r}))^{l_4}\ldots(\bar{a}\bar{r}+\bar{d}(\bar{r})^{l_k})^n = \bar{0}$. It is obvious that $\overline{U}$ is prime. Therefore by Corollary 2.3, we have either $\overline{U}$ is commutative or $\bar{d} = 0$, that is either $d(U) \subseteq MU$ or $[U, U] \subset MU$. Hence $d(U)[U, U] \subseteq MU$, where MU runs over all prime ideals of U. Since $\cap_M MU = 0$, we obtain $d(U)[U, U] = 0$.

By using the theory of orthogonal completion for semiprime rings (see [1], Chapter 3), it is clear that there exists a central idempotent element e in U such that on the direct sum decomposition $R = eU \oplus (1-e)U$, d vanishes identically on eU and the ring $(1-e)U$ is commutative. This completes the proof of the theorem. $\square$

Theorem 2.5. *Let A be a 2-torsion-free non-commutative Banach algebra with Jacobson radical $rad(A)$. Let $F = L_a + d$ be a continuous generalized derivation of R, where L_a denote the left multiplication by some element $a \in A$ and d is the associated derivation of A. If $(d(r)^{l_1}F(r)^{l_2}d(r)^{l_3}F(r)^{l_4}\ldots F(r)^{l_k})^n \in rad(A)$ for all $r \in A$, where $l_1, l_2, \ldots, l_k$ are fixed non-negative integers not all zero, and n is fixed positive integer, then $d(A) \subseteq rad(A)$.*

Proof. By the hypothesis F is continuous and moreover since it is well-known that L_a also continuous, we get that d is continuous. In [4], Sinclair proved that any continuous derivation of a Banach algebra leaves the primitive ideals invariant. Hence, for any primitive ideal P of A, it is obvious that $F(P) \subseteq aP + d(P) \subseteq P$. It means that the continuous generalized derivation F leaves the primitive ideals invariant. Denote $A/P = \overline{A}$ for any primitive ideals P. Thus we can define the generalized derivation $F_P : \overline{A} \to \overline{A}$ by $F_P(\bar{x}) = F_P(x + P) = F(x) + P = ax + d(x) + P = ax + d_P(\bar{x})$ for all $\bar{x} \in \overline{A}$, where $A/P = \overline{A}$ is a factor Banach algebra. Since P is a primitive ideal, the factor algebra $\overline{A}$ is primitive and so it is prime. The hypothesis $(d(r)^{l_1} F(r)^{l_2} d(r)^{l_3} F(r)^{l_4} \ldots F(r)^{l_k})^n \in rad(A)$ for all $r \in A$, yields that $(d_P(\bar{r})^{l_1} F_P(\bar{r})^{l_2} (d_P(\bar{r})^{l_3} F_P(\bar{r})^{l_4} \ldots F_P(\bar{r})^{l_k})^n = 0$ for all $\bar{r} \in \overline{A}$. From Corollary 2.3, it is immediate that either $\overline{A}$ is commutative or $d = \bar{0}$, that is $[A, A] \subseteq P$ or $d(A) \subseteq P$.

Now we assume that P is a primitive ideal such that $\overline{A}$ is commutative. In [2] Singer and Werner proved that any continuous linear derivation on a commutative Banach algebra maps the algebra into the radical. Furthermore by a result of Jonhson and Sinclair [23], any linear derivation on semisimple Banach algebra is continuous. We know that there are no nonzero linear continuous derivations on commutative semisimple Banach algebras. Therefore $d = \bar{0}$ in $\overline{A}$.

Hence in any case we get $d(A) \subseteq P$ for all primitive ideal P of A. Since $rad(A)$ is the intersection of all primitive ideals, we get $d(A) \subseteq rad(A)$, we get the required conclusion. □

Acknowledgments

The author is greatly indebted to the referee for her/his useful suggestions. This research was supported by the Natural Science Research Foundation of Anhui Provincial Education Department (No. KJ2012B125) and also by the Anhui Province College Excellent Young Talents Fund Project (Nos. 2012SQRL155; 2012SQRL156) of China.

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